Regarding the Deceleration of the Universe

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In the standard big bang model, the expansion rate of the Universe is predicted to slow down as the Universe evolves.[1,2] The deceleration parameter, q_0, measures the present deceleration rate. The deceleration causes a deviation from the linear relation between distance and red shift (the Hubble law) at large red shifts. A proposed approach for determining q_0 is to measure the red shifts and distances of Type Ia supernovae,[3,4] whose luminosities may be calibrated so that the inverse-square-law can be used to judge their distance. To date, the strategy has been to use measurements at low red shift $z \leq 0.1$ to precisely determine the linear distance-red shift relation[5] and then focus on measuring the deviation from the linear law[3,4] using supernovae at high red shifts $z = 0.35 - 0.6$ to determine q_0.

This note makes a simple point that is important in interpreting the supernovae results: At red shift $z = 0.35 - 0.6$, the deviation from the linear Hubble relation does not depend on q_0 alone, but also on the matter-energy content of the universe. It is well-known that this dependence on matter-energy content is significant at sufficiently high red shifts, but it has not been generally appreciated that the dependence is non-negligible at red shifts as modest as $z = 0.35$.

The effect is easy to understand. Observations at non-zero red shift measure properties of the universe at earlier times when the light was emitted. In some models, such as open models with low matter density, the deceleration rate at $z \approx 0.5$ is comparable to the deceleration rate today (q_0). In other models, such as those with non-zero cosmological constant, the universe is undergoing a fairly rapid transition from a decelerating, matter-dominated universe at $z = 0.35$ towards an accelerated expansion in a universe dominated by a non-zero cosmological constant. The two models predict a different deviation from the linear Hubble law at $z = 0.35$ even if the present deceleration rate q_0 is the same.

The "luminosity distance" between a given source and us is defined as $d_L^2 \equiv \mathcal{L}/4\pi\mathcal{F}$ where $\mathcal{L}$ is the emitted energy per unit time and $\mathcal{F}$ is the energy received per unit time. An

elementary calculation[1,2] shows that $d_L = (1+z)r_1$ where the comoving distance r_1 satisfies:

$$\int_0^{r_1} \frac{dr}{(1-kr)^{1/2}} = \int_0^z \frac{dz'\,[H_0(1+z')]^{-1}}{[\Omega_m(1+z') + \Omega_\Lambda(1+z')^{-2} + \Omega_\alpha(1+z)^{1+3\alpha} + (1-\Omega_m-\Omega_\Lambda-\Omega_\alpha)]^{-1/2}};$$

(1)

here H_0 is the Hubble constant, and Ω_m, Ω_Λ and Ω_α are the ratios of the matter density, the cosmological constant, and other possible forms of matter-energy, respectively, to the critical density. We have generalized to include possible matter-energy with general equation-of-state $\alpha \equiv p/\rho$, where p is the pressure and ρ is the energy density. Then, $q_0 = \frac{1}{2}\Omega_m - \Omega_\Lambda + \left(\frac{1+3\alpha}{2}\right)\Omega_\alpha$.

Figures 1 and 2 illustrate the point. Figure 1 shows the distance-red shift relation for three different models. Note that the curves diverge from one another before $z = 0.35$ even though they correspond to identical H_0 and q_0. Figure 2 is a blow-up of the range $z = 0.35 - 0.6$ expressed in terms of apparent magnitude ($m = 5\log[d_L] + $ constant) vs. red shift. The Figure magnifies the differences among the models in Figure 1, and shows a model with $q_0 = -0.25$ and $\Omega_\Lambda = \Omega_m = 0.5$ that is nearly indistinguishable from an open model with $q_0 \approx 0$.

Although the current strategy does not measure q_0 alone, it provides a useful constraint on a combination of Ω_m, Ω_Λ, Ω_α and the spatial curvature. If one imagines a multi-dimensional parameter-space with axes corresponding to each of these variables, a precise measurement at any given z constrains models to some swath which depends on z. Goobar and Perlmutter[6] have illustrated this for models with $\Omega_\alpha = 0$, and it is straightforward to generalize to other models. By precise measurements at disparate values of z, especially in the range 0.2 to 0.5, a set of constraints can be obtained which may make it possible to discriminate q_0 and the other parameters independently.

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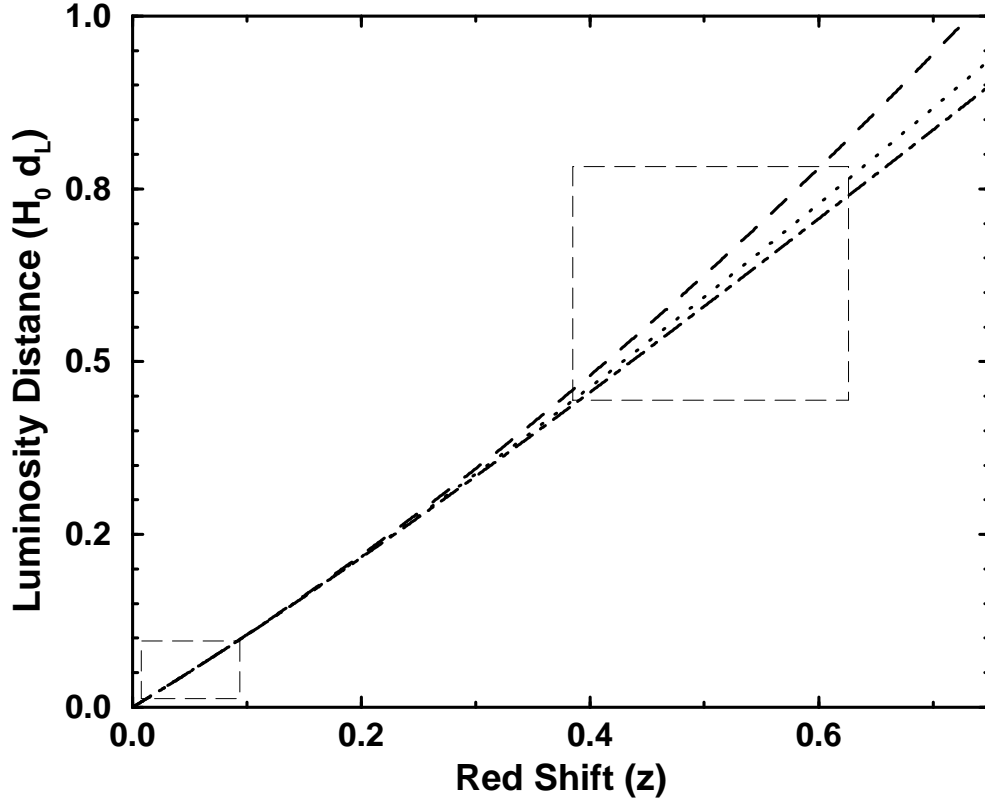[6] Goobar, A. and Perlmutter, S., *Ap. J.* 450, 15 (1995).

FIG. 1. A demonstration that the luminosity distance vs. red shift curves can differ significantly at $z \approx 0.5$ (upper box) for models with same $q_0 = 0$: (a) matter-dominated model (dashed) with $\Omega_m \rightarrow 0$, $\Omega_\Lambda = \Omega_\alpha = 0$; (b) $\Omega_m = 2/3$, $\Omega_\Lambda = 1/3$ and $\Omega_\alpha = 0$ (dotted); and (c) $\Omega_m = 0.2$; $\Omega_\Lambda = 0.45$, and $\Omega_\alpha = .35$ (dot-dashed) with $\alpha = 1/3$ representing hot, relativistic matter.
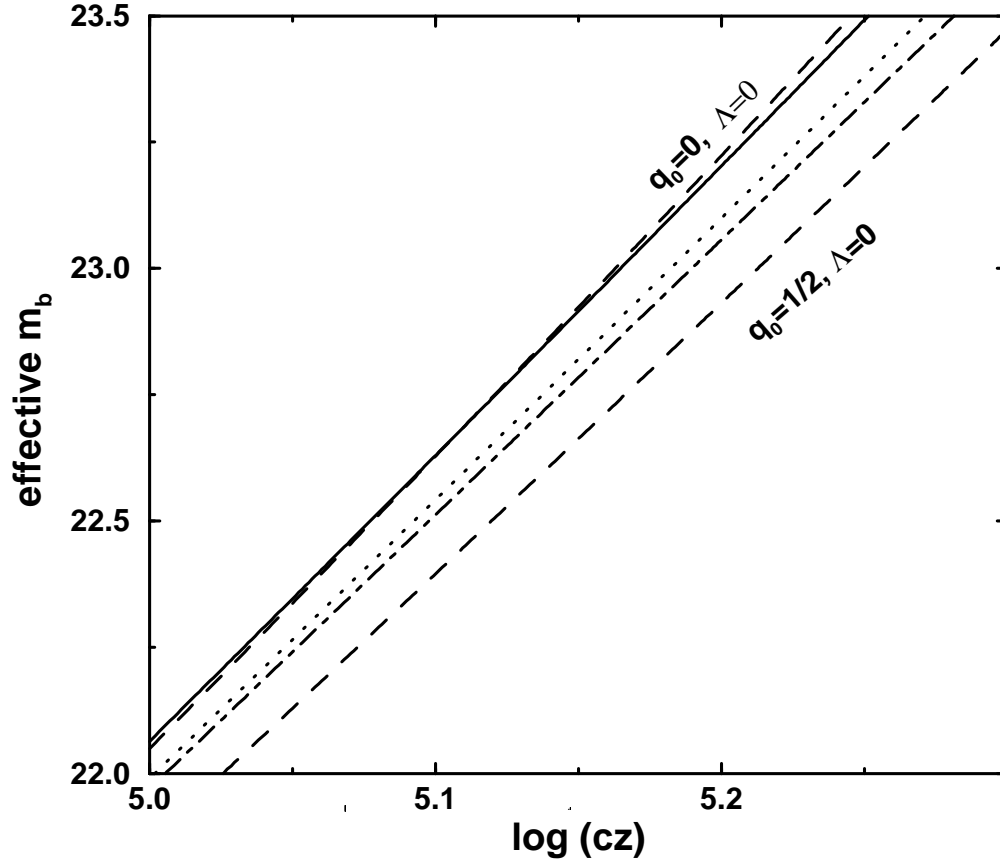
FIG. 2. The red shift regime $z = 0.35 - 0.6$ replotted in terms of effective apparent magnitude. The upper and lower dashed curves correspond to $q_0 = 0$ and $q_0 = 1/2$ assuming $\Omega_\Lambda = \Omega_\alpha = 0$. The dotted and dot-dashed curves (same as Figure 1) also have $q_0 = 0$, but differ significantly from the upper dashed curve. The solid line corresponds to $\Omega_\Lambda = \Omega_m = 0.5$ and $q_0 = -0.25$, yet it is difficult to distinguish from the upper dashed curve with $q_0 = 0$.